**Signals & Systems**

**EEE-223**

Lab # 10



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**LAB # 10**

**Trigonometric (Real) Fourier Series Representation and its Properties**

**Lab 10-** **Trigonometric (Real) Fourier Series Representation and its Properties**

**Pre-Lab Tasks**

**10.1 Trigonometric Fourier Series:**

A second form of Fourier series is introduced in this section. Suppose that a signal is defined in the time interval. Then, by using the trigonometric Fourier series, can be expressed in time interval as a sum of sinusoidal signals, namely, sines and cosines, where each signal has frequency rad/s.

The mathematical expression (equation 10.1) is



The coefficients, , , . . ., ,, . . . of the trigonometric Fourier series are computed by

 

 

 

**Example:**

The signal that will be expanded is the same signal used at the previous example. Thus, the problem is to expand in trigonometric Fourier series the signal.

First the trigonometric Fourier coefficients,  and the dc component  are computed according to equations 10.3, 10.4 and 10.2, respectively, for. Next, is approximated according to the relationship (equation 10.5)



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| Commands | Results/Comments |
| T=3;  t0=0;  w=2\*pi/T;  syms t  x=exp(-t); | Definition of and of the signal |
| a0=(1/T)\*int(x,t,t0,t0+T);  for n=1:200  b(n)=(2/T)\*int(x\*cos(n\*w\*t),t,t0,t0+T);  end  for n=1:200  c(n)=(2/T)\*int(x\*sin(n\*w\*t),t,t0,t0+T);  end | Computation of trigonometric coefficients according to equations 10.2, 10.3 and 10.4. |
| k=1:200;  xx=a0+sum(b.\*cos(k\*w\*t))+sum(c.\*sin(k\*w\*t)) | The signal is approximated by equation 10.1 or more precisely from equation 10.5 with N=200. |
| ezplot(xx, [t0 t0+T]);  title('Approximation with 201 terms') | Graph of the approximate signal that was computed by the terms of the trigonometric Fourier series.  lab101.bmp  The approximation with 201 terms of the original signal is very good. |

In order to understand the importance of the number terms used for the approximation of the original signal , the approximate signal is constructed for different values of . Approximation with five terms ()

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| Commands | Results/Comments |
| clear b c  for n=1:5  b(n)=(2/T)\*int(x\*cos(n\*w\*t),t,t0,t0+T);  c(n)=(2/T)\*int(x\*sin(n\*w\*t),t,t0,t0+T);  end  k=1:5;  xx=a0+sum(b.\*cos(k\*w\*t))+sum(c.\*sin(k\*w\*t))  ezplot(xx, [t0 t0+T]);  title('Approximation with 6 terms') | When the signal is approximated with 5 terms, it is pretty dissimilar to the original signal.  lab102.bmp |

Approximation with 20 terms ()

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| Commands | Results/Comments |
| for n=1:20  b(n)=(2/T)\*int(x\*cos(n\*w\*t),t,t0,t0+T);  c(n)=(2/T)\*int(x\*sin(n\*w\*t),t,t0,t0+T);  end  k=1:20;  xx=a0+sum(b.\*cos(k\*w\*t)) +sum(c.\*sin(k\*w\*t));  ezplot(xx, [t0 t0+T]);  title('Approximation with 21 terms') | The approximated signal is now approximated by 20 terms and it is quite similar to the original signal. The approximation is quite satisfactory.  lab103.bmp |

**10.2 Properties of Fourier Series:**

**10.2.1 Linearity**

Suppose that the complex exponential Fourier series coefficients of the periodic signals  and are denoted by and, respectively, or In other words and . Moreover, let ,denote two complex numbers. Then



To verify the linearity property, we consider the periodic signals , and the scalars and .

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| Commands | Results/Comments |
| t0=0;  T=2\*pi;  w=2\*pi/T;  syms t  z1=3+2i; z2=2;  x=cos(t); y=sin(2\*t);  f=z1\*x+z2\*y;  k=-5:5;  left=(1/T)\*int(f\*exp(-j\*k\*w\*t),t,t0,t0+T);  left=eval(left);  subplot(211);  stem(k,abs(left));  legend('Magnitude');  title('Coefficients of the left part');  subplot(212);  stem(k,angle(left));  legend('Angle'); | First we determine the complex exponential Fourier series coefficients of the left part; that is, we compute the coefficients of the signal . The period of is  lab104.bmp |
| a=(1/T)\*int(x\*exp(-j\*k\*w\*t),t,t0,t0+T);  b=(1/T)\*int(y\*exp(-j\*k\*w\*t),t,t0,t0+T);  right=z1\*a+z2\*b;  subplot(211);  right=eval(right);  stem(k,abs(right));  legend('Magnitude');  title('Coefficients of the right part');  subplot(212);  stem(k,angle(right));  legend('Angle'); | In order to derive the right part of the linearity equation, first coefficients are computed and formulate the right part of the linearity equation.  lab105.bmp |

The two graphs are identical; thus the linearity property is verified.

**10.2.2 Time Shifting**

A shift in time of the periodic signal results on a phase change of the Fourier series coefficients. So, if , the exact relationship is



In order to verify time shifting property, we consider the periodic signal that in one period is given by . Moreover, we set . Consequently the signal is given by .

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| Commands | Results/Comments |
| t0=0;  T=10;  w=2\*pi/T;  syms t  x=t\*exp(-5\*t)  k=-5:5;  a=(1/T)\*int(x\*exp(-j\*k\*w\*t),t,t0,t0+T);  a1=eval(a);  subplot(211);  stem(k,abs(a1));  title(' Coefficients of x(t)=te^-^5^t');  legend('Magnitude');  subplot(212);  stem(k,angle(a1));  legend('Angle'); | First the Fourier series coefficients for the given signal are computed and plotted.  lab106.bmp |
| t1=3;  right= exp(-j\*k\*w\*t1).\*a;  right =eval(right);  subplot(211);  stem(k,abs(right));  legend('Magnitude');  title('Right part');  subplot(212);  stem(k,angle(right));  legend('Angle'); | Next, the right part of the time shifting equation is computed.  lab107.bmp |
| x=(t-t1).\*exp(-5\*(t-t1));  a=(1/T)\*int(x\*exp(-j\*k\*w\*t),t,t0+t1,t0+T+t1);  coe=eval(a);  subplot(211);  stem(k,abs(coe));  legend('Magnitude');  title(' Coefficient of (t-3)exp(-5(t-3)) ');  subplot(212);  stem(k,angle(coe));  legend('Angle'); | Finally, the time shifted version of is defined, i.e.,, and corresponding Fourier series coefficients are computed.  lab108.bmp |

The two last graphs are identical; hence, the time shift property is confirmed. Comparing the two last graphs with the first one, we notice that indeed the magnitude does not change, but the phase is different.

**10.2.3 Time Reversal**

The Fourier series coefficients of the reflected version of a signal are also a reflection of the coefficients of . So, if , the mathematical expression is



In order to validate the time reversal property, we consider the periodic signal that in one period is given by .

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| Commands | Results/Comments |
| t0=0;  T=2\*pi;  w=2\*pi/T;  syms t  x=t\*cos(t) ;  k=-5:5;  a=(1/T)\*int(x\*exp(-j\*k\*w\*t),t,t0,t0+T);  a1=eval(a);  subplot(211);  stem(k,real(a1));  legend('Re[a\_k]');  title('Coefficients of x(t)');  subplot(212);  stem(k,imag (a1));  legend('Im[a\_k]'); | Fourier series coefficients are computed and plotted.  lab109.bmp |
| x\_=-t\*cos(-t) ;  b=(1/T)\*int(x\_\*exp(-j\*k\*w\*t),t,t0-T,t0);  b1=eval(b)  subplot(211);  stem(k,real(b1));  legend('Re[b\_k]');  title(' Coefficients of x(-t)');  subplot(212);  stem(k,imag (b1));  legend('Im[b\_k]'); | Next the coefficients are computed for the time reversed version , and we notice that . Hence the time reversal property is confirmed.  lab1010.bmp |

**10.2.4 Time Scaling**

The Fourier series coefficients of a time scaled version and do not change. On the other hand, the fundamental period of the time scaled version becomes , and the fundamental frequency becomes . The mathematical expression is



The time scaling property is confirmed by using the periodic signal that in one period is given by .

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| Commands | Results/Comments |
| syms t  t0=0;  T=2\*pi;  w=2\*pi/T;  x=t\*cos(t) ;  k=-5:5;  a=(1/T)\*int(x\*exp(-j\*k\*w\*t),t,t0,t0+T);  a1=eval(a)  subplot(211);  stem(k,abs(a1));  legend('Magnitude');  title(' Coefficients of x(t)');  subplot(212);  stem(k,angle(a1));  legend('Angle'); | First the Fourier series exponential components for the signal  are computed and plotted.  lab1011.bmp |
| lamda=2;  T=T/ lamda ;  w=2\*pi/T;  x= lamda \*t\*cos(lamda \*t) ;  k=-5:5;  a=(1/T)\*int(x\*exp(-j\*k\*w\*t),t,t0,t0+T);  a1=eval(a)  subplot(211);  stem(k,abs(a1));  legend('Magnitude');  title(' Coefficients of x(2t)');  subplot(212);  stem(k,angle(a1));  legend('Angle'); | Next the coefficients of the time scaled signal , and it is seen that . Hence the time scaling property is confirmed.  lab1012.bmp |

**10.2.5 Signal Multiplication**

The Fourier series coefficient of the product of two signals equals the convolution of the Fourier series coefficients of each signal. Suppose that  and , we have 10. 6 as



Where \* denotes discrete time convolution. To verify property 10.6, we consider the signalsand .

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| Commands | Results/Comments |
| syms t  t0=0;  T=2\*pi;  w=2\*pi/T;  x=cos(t) ;  k=-5:5;  a=(1/T)\*int(x\*exp(-j\*k\*w\*t),t,t0,t0+T);  a1=eval(a);  y=sin(t);  b=(1/T)\*int(y\*exp(-j\*k\*w\*t),t,t0,t0+T);  b1=eval(b);  left=conv(a1,b1);  subplot(211);  stem(-10:10,abs(left));  legend('Magnitude');  title(' a\_k\*b\_k');  subplot(212);  stem(-10:10,angle(left));  legend('Angle'); | First, the exponential Fourier series coefficients and and their convolution is computed. Notice that the convolution is implemented between two complex valued sequences.  lab1013.bmp |
| z=x\*y;  k=-10:10;  c=(1/T)\*int(z\*exp(-j\*k\*w\*t),t,t0,t0+T);  c1=eval(c)  subplot(211);  stem(k,abs(c1));  legend('Magnitude');  title(' Coefficients of x(t)y(t)');  subplot(212);  stem(k,angle(c1));  legend('Angle'); | Next, the Fourier series coefficients are computed for the signal .  lab1014.bmp |

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**In-Lab Tasks**

**Task 01: The periodic signal is defined in one period as . Plot approximate signal using 81 terms of trigonometric form of Fourier series.**

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| T = 6; %Time Period  t0 = 0;  w = 2\*pi/T; %Angular Frequency    syms t %t as symbol declaration  x = t.\*exp(-t);    a0 = (1/T)\*int(x,t,t0,t0+T);  for n = 1:80  b(n) = (2/T)\*int(x\*cos(n\*w\*t),t,t0,t0+T);  end  for n = 1:80  c(n) = (2/T)\*int(x\*sin(n\*w\*t),t,t0,t0+T);  end  k = 1:80;  xx = a0+sum(b.\*cos(k\*w\*t))+sum(c.\*sin(k\*w\*t))  ezplot(xx, [t0 t0+T]);  title('Approximation with 81 terms')  Chart  Description automatically generated |

**Task 02: Plot the coefficients of the trigonometric Fourier series for the periodic signal that in one period is defined by .**

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| T = 6; %Time Period  t0 = -3;  w = 2\*pi/T; %Angular Frequency    syms t %t as symbol declaration  x = exp(-t.^2);  k = -3:3;  a0 = (1/T)\*int(x,t,t0,t0+T);  b = (2/T)\*int(x\*cos(k\*w\*t),t,t0,t0+T);  c = (2/T)\*int(x\*sin(k\*w\*t),t,t0,t0+T);    figure  subplot(3,1,1)  stem(0,a0),legend('a0'),title('Trignometric Coefficients of x(t)');  subplot(3,1,2)  stem(k,b),legend('b\_k');  subplot(3,1,3)  stem(k,c),legend('c\_k');  Graphical user interface  Description automatically generated with medium confidence |

**Task 03: The periodic signal in a period is given by**

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**Plot in one period the approximate signals using 41 and 201 term of the trigonometric Fourier series. Furthermore, each time plot the complex exponential coefficients.**

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| t0 = 0;  T = 2;  w = 2\*pi/T;  syms t  x=heaviside(t)-heaviside(t-1);  a0 = (1/T)\*int(x,t,t0,t0+T);  for n = 1:40 %Approximation using 41 terms  b(n) = (2/T)\*int(x\*cos(n\*w\*t),t,t0,t0+T);  end  for n = 1:40 %Approximation using 41 terms  c(n) = (2/T)\*int(x\*sin(n\*w\*t),t,t0,t0+T);  end    k = 1:40; %Approximation using 41 terms  xx = a0 + sum(b.\*cos(k\*w\*t)) + sum(c.\*sin(k\*w\*t));  ezplot(xx, [t0 t0+T]);  title('Approximation with 41 terms')  Graphical user interface, chart  Description automatically generated  t0 = 0;  T = 2;  w = 2\*pi/T;  syms t  x=heaviside(t)-heaviside(t-1);  a0 = (1/T)\*int(x,t,t0,t0+T);  for n = 1:200  b(n) = (2/T)\*int(x\*cos(n\*w\*t),t,t0,t0+T);  c(n) = (2/T)\*int(x\*sin(n\*w\*t),t,t0,t0+T);  end  k = 1:200;  xx = a0 + sum(b.\*cos(k\*w\*t)) + sum(c.\*sin(k\*w\*t));  ezplot(xx, [t0 t0+T]);  title('Approximation with 201 terms')  Graphical user interface, chart  Description automatically generated |

**Task 04: The periodic signal in a period is given by**

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**Calculate the approximation percentage when the signal is approximated by 3, 5, 7, and 17 terms of the trigonometric Fourier series. Furthermore, plot the signal in each case.**

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| T = 2;  t0 = 0;  w = 2\*pi/T;  syms t  x = heaviside(t)+((heaviside(t-1)).\*(1-t));  ezplot(x,[t0 t0+T]),grid on  Chart, line chart  Description automatically generated  a0 = (1/T)\*int(x,t,t0,t0+T);  for n = 1:2  b(n) = (2/T)\*int(x\*cos(n\*w\*t),t,t0,t0+T);  c(n) = (2/T)\*int(x\*sin(n\*w\*t),t,t0,t0+T);  end  k = 1:2;  xx1 = a0 + sum(b.\*cos(k\*w\*t)) + sum(c.\*sin(k\*w\*t));  ezplot(xx1, [t0 t0+T]);  title('Approximation with 3 terms')  Chart  Description automatically generated  a0 = (1/T)\*int(x,t,t0,t0+T);  for n = 1:4  b(n) = (2/T)\*int(x\*cos(n\*w\*t),t,t0,t0+T);  c(n) = (2/T)\*int(x\*sin(n\*w\*t),t,t0,t0+T);  end  k = 1:4;  xx1 = a0 + sum(b.\*cos(k\*w\*t)) + sum(c.\*sin(k\*w\*t));  ezplot(xx1, [t0 t0+T]);  title('Approximation with 5 terms')  Graphical user interface, chart  Description automatically generated    a0 = (1/T)\*int(x,t,t0,t0+T);  for n = 1:6  b(n) = (2/T)\*int(x\*cos(n\*w\*t),t,t0,t0+T);  c(n) = (2/T)\*int(x\*sin(n\*w\*t),t,t0,t0+T);  end  k = 1:6;  xx1 = a0 + sum(b.\*cos(k\*w\*t)) + sum(c.\*sin(k\*w\*t));  ezplot(xx1, [t0 t0+T]);  title('Approximation with 7 terms')  Graphical user interface, chart  Description automatically generated  a0 = (1/T)\*int(x,t,t0,t0+T);  for n = 1:16  b(n) = (2/T)\*int(x\*cos(n\*w\*t),t,t0,t0+T);  c(n) = (2/T)\*int(x\*sin(n\*w\*t),t,t0,t0+T);  end  k = 1:16;  xx1 = a0 + sum(b.\*cos(k\*w\*t)) + sum(c.\*sin(k\*w\*t));  ezplot(xx1, [t0 t0+T]);  title('Approximation with 17 terms')  Graphical user interface, chart  Description automatically generated |

**Post-Lab Task**

## Critical Analysis / Conclusion

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| In this lab, we learnt how to plot signals with approximations and observed the effect with increased number of terms and how to find coefficients “a\_k” trigonometric Fourier series in MATLAB. Moreover, we plotted and observed the coefficients and angle of trigonometric exponential Fourier series. |

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| **Lab Assessment** | | |
| **Pre-Lab** | **/1** | **/10** |
| **In-Lab** | **/5** |
| **Critical Analysis** | **/4** |
| **Instructor Signature and Comments** | | |